Maneuvering assistant for truck and trailer combinations with arbitrary trailer hitching

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Abstract—Stabilizing controllers can assist the driver to safely maneuver of truck and trailer combinations like the EuroCombis. This paper presents a novel nonlinear controller for combinations of a nonholonomic tractor car with two semi-trailers based upon an instantaneous approximation of the vehicle kinematics, an exact input-state linearization technique and a model-reference control scheme. The main advantage of the suggested controller is its universality, as it can be applied to truck and trailer combinations with an arbitrary hitching offset as well as in both driving directions. Convergence and robustness properties of the proposed controller have been validated by a series of simulations and experiments with a combination, which possesses one off-axle and one on-axle joint. The presented method can also be extended to other truck and trailer combinations.

Keywords - truck and trailer combination, off-axle hitching, input-state linearization, model-reference control scheme

I. INTRODUCTION

Automatic and autonomous transportation systems are the objective of numerous research initiatives at present. Among those the controllability of articulated multi-body vehicles plays a prominent role. The major control challenge in this type of plant arises mainly from the nonlinear nature of their kinematics and its instability in the case of backward motion in a conjunction with the steering limitations.

The introduction of the European Council Directive 96/53/EG in 1996 created a legal basis within the European Union for truck and trailer combinations up to lengths of 25.25 m and up to a total weight of 60 tons. As a result of the developments in the logistics industry different truck and trailer configurations emerged, which are known as “EuroCombis”, see Fig. 1. Most of those EuroCombis (variants A - E) feature two hitching joints and can be regarded kinematically as a tractor car with two semi-trailers. However, these EuroCombis differ from each other with regard to their trailer hitching offsets. This is a crucial point which leads to partially different behavior of the vehicles. The ability to handle systems with an arbitrary type of trailer hitching is therefore of utmost importance for designing a universal maneuvering assistant for the most relevant truck and trailer combinations with two semi-trailers (TTC).

Through the many years of international research numerous stabilizing controllers for articulated vehicles were developed, depending on the existence and direction of the trailer hitching offset. The earlier works, based on the method of exact input-state linearization, were limited to the case, that the hitching point was located directly on the center of the rear axle of the preceding towing unit (so-called “on-axle” hitching) [2], [3], [4], [5]. It was also noted that the exact input-state linearization is not applicable for multi-body articulated vehicles with an offset in the trailer hitching (so-called “off-axle” hitching) [6], [7]. In [7] an approximation of the system with a hitching offset at the truck by an on-axle system was suggested. The approximated “ghost vehicle” provided the same steady state behavior and allowed an exact input-state linearization. During path-tracking the modeling inaccuracies were considered as disturbances.

Similar results were later achieved for train-like vehicles with a hitching offset by applying the method of exact input-output linearization instead of input-state linearization [8], [9]. The peculiarity of these control laws is the dependence of the choice of the guide-point and therefore the controller structure has to be adjusted based upon the driving direction.

It should be noted that most works on the control of articulated vehicles are limited to a negative hitching offset (joint behind the rear axle of the preceding unit), while to the present time there are relatively few research results available regarding the control of train-like vehicles with positive (or an arbitrary) hitching offset. The reason are the mentioned difficulties to apply the method of exact linearization. In [10] this problem was avoided by “conventional” (Jacobian) linearization. Promising results provides the application of the Lyapunov method to design a controller. In [11] this is shown only for a vehicle with a single semi-trailer. In addition the application of known approaches to the path-tracking problem for train-like vehicles with off-axle hitching provides quite complex control laws and complicates their practical application.
The measures proposed in this paper for TTC with arbitrary trailer hitching differ from previous approaches by the underlying hypothesis that the stabilization and the path-tracking tasks can be treated separately, resulting in a cascaded control structure. The novelty of the proposed lower-level stabilizing control loop, which forms a maneuvering assistant, arises from applying an instantaneous approximation of the vehicle kinematics, an exact input-state linearization technique and a model-reference control scheme. This approach offers all the advantages of a cascade control structure and will allow at a later time the design of simpler control laws of a superimposed path-tracking controller.

II. STABILIZING CONTROLLER

A. Analysis of possible solutions

The nonlinear control theory provides a variety of methods for the analysis and synthesis of nonlinear control systems. The most successful method applied to truck and trailer combinations is still the exact feedback linearization. The special attractiveness of this method is explained by the elegant way, in which the requirements of stability and control performance can be addressed using the well-known methods of linear control theory to the linearized system. There are two well-known exact linearization techniques, input-output linearization (IOL) and input-state linearization (ISL), applicable for the class of nonlinear affine in control systems. The application of IOL is usually more straightforward than that of ISL. However, a control law based on IOL is only suitable with a stable internal dynamics of the controlled system. In case of IOL of a truck and trailer combination the stability of the internal dynamics as shown in [8] depends of the direction of motion, the sign of the hitching offset and a choice of the guide-point of the vehicle. Since the ISL produces no internal dynamics in the controlled system, it would allow to design the universal control laws valid for truck and trailer combinations with an arbitrary hitching offset as well as independent of the direction of motion. Unfortunately, the analytical determination of the flat output for the nonlinear system, like a TTC, can be an unachievable task. Therefore, irregular approaches to simplify the task may be required.

It is well-known, that a substitution of variables in nonlinear differential equations of the vehicle’s model provides an affine in control system without increasing the order of the system. Our investigation has shown, that a further instantaneous approximation of a single term in the equations allows to obtain the solution of a partial differential equation leading to a flat output of the approximated system with relative ease. Based upon this approach, the state as well the input transformation can be found. After such an approximate input-state linearization the design of a linear state regulator for the stabilizing and tracking controller (maneuvering assistant) can be done. To provide robust performance under the influence of model inaccuracies and disturbances a model-reference control scheme is used.

B. Kinematic model of the vehicle with two semi-trailers

The design of a controller requires a model of the TTC that describes the system behavior in forward and reverse direction of movements with sufficient accuracy. A purely kinematic model can be established, which neglects dynamic effects (e.g. inertia and slip) and reduces to a planar single-track model. The kinematic modeling leads to a much simpler and better interpretable system of differential equations, which reproduces the processes in truck and trailer combinations with reasonable accuracy.

Fig. 2 depicts the schematic representation of a TTC. The first joint is assumed to be an off-axle joint with an arbitrary hitching offset (pictured positive) while the second hitching is on-axle. The following variables and parameters are illustrated in Fig. 2:

- \( \phi_0(t) \) - steering angle
- \( \phi_1(t) \) - angle between first semi-trailer and tractor
- \( \phi_2(t) \) - angle between second and first semi-trailer
- \( \lambda \) - distance traveled along the path
- \( V_0(t) \) - tractors’ front axle speed along the path
- \( V_1(t) \) - speed of first semi-trailer
- \( l_0 \) - distance between tractors’ axles
- \( d_1 \) - hitching offset (positive towards tractors’ front axle)
- \( l_1 \) - length of first semi-trailer
- \( l_2 \) - length of second semi-trailer

For the design of a stabilizing controller only the behavior of the internal degrees of freedom \( \phi_1(t) \) and \( \phi_2(t) \) have to be considered. For ease of analysis and design of the control system, the following notations were introduced:

- state variables: \( x_1 = \phi_2, x_2 = \phi_1 \);
- control variables: \( x_3 = \phi_0, V_0 \);
- model parameters: \( a_1 = V_0 \frac{1}{l_2}, a_2 = V_0 \frac{d_1}{l_2}, a_3 = V_0 \frac{1}{l_1}, a_4 = V_0 \frac{d_1}{l_3}, a_5 = V_0 \frac{1}{l_0} \);
- short form of the functions: \( c_i = \cos x_i, s_i = \sin x_i \).
Under these notations the behavior of the internal coordinates \( \phi_2(t) \) and \( \phi_1(t) \) without consideration of external degrees of freedom leads to a system of two nonlinear differential equations:

\[
\begin{align*}
\dot{x}_1 &= -a_1 x_1 c_2 s_3 - a_2 x_1 s_1 s_3 - a_3 x_1 c_4 s_3 - a_4 x_1 c_2 s_3 \\
\dot{x}_2 &= -a_1 x_2 c_3 - a_2 x_2 s_1 s_3 - a_3 x_2 c_4 s_3 - a_4 x_2 c_2 s_3 \\
\end{align*}
\]

(1)

ISL is applicable to a class of nonlinear affine in control systems, whose origin is an equilibrium with zero control:

\[
\dot{x} = f(\bar{x}) + g(\bar{x}) \cdot u, \quad \bar{x} \in \mathbb{R}^n, \quad u \in \mathbb{R},
\]

with \( f(\bar{x}) \) and \( g(\bar{x}) \) smooth \( n \times 1 \) vector functions.

To convert the model (1) into (2), the speed of the first semi-trailer has to be used instead of the tractors’ front axle speed. This is equivalent to introducing the following variable substitution:

\[
V_1 = V_0 \cdot c_2 \cdot c_4.
\]

(3)

Further, after another substitution

\[
u = \tan x_3
\]

(4)

the functions of the affine in control plant model (2) become:

\[
\begin{align*}
f(\bar{x}) &= \begin{bmatrix} f_1(\bar{x}) \\ f_2(\bar{x}) \end{bmatrix} = \begin{bmatrix} -q_1 s_1 + q_3 \tan x_2 \\ -q_4 \tan x_2 \end{bmatrix}, \\
g(\bar{x}) &= \begin{bmatrix} g_1(\bar{x}) \\ g_2(\bar{x}) \end{bmatrix} = \begin{bmatrix} -q_2 s_1 \tan x_2 - q_4 \\ q_4 - q_5 \end{bmatrix}. \\
\end{align*}
\]

(5)

The model parameters \( q_i \) correspond to the parameters \( a_i \), if the speed \( V_1 \) replaces \( V_0 \). This means, that the speed \( V_1 \) must be kept constant by the tractor’s speed control according to (3). The speed \( V_0 \) can be completely eliminated from the differential equations to avoid singularities at \( V_0 = 0 \), if the time derivatives are replaced by the derivatives with respect to a new independent variable \( \lambda \), e.g. by means of the so-called time scaling [12].

C. Approximation and input-state linearization

ISL is achieved by a conversion from a system of nonlinear differential equations (2) to the canonical Brunovsky form (13):

\[
\begin{align*}
\dot{z}_1 &= z_2, \\
\dot{z}_2 &= z_3, \\
\vdots &
\end{align*}
\]

by a linearizing feedback, consisting of:

- diffeomorphic state transformation \( \bar{z}(\bar{x}) = [z_1(\bar{x}) \ldots z_n(\bar{x})]^T \)
- input transformation \( u = \frac{1}{\beta(\bar{z})} (v - \alpha(\bar{z})) \), with \( v \) the new linearized input.

The plant in this case is covered by the feedback on theoretically all state variables. From this follows the definition of the feedback linearization. There are no internal dynamics, because all state variables are linearized by ISL. The principal difference from the “conventional” (Jacobian) linearization is that the exact linearization is not approximate but an equivalent transformation.

The plant (2), (5) is controllable in the vicinity of the origin, where controllability matrix \( Q_c = [ad_{ij} \bar{g} \ldots ad_{ij}^{n-1} \bar{g}] \) has full rank. It is easy to obtain the controllability conditions in the origin: \( q_4 \neq q_5 \) and \( q_1q_4 \neq q_3q_5 \), which can be summarized into constraints on the vehicle’s geometry: \( d_1 \neq l_1 \) and \( d_1 \neq l_2 \). In truck and trailer combinations the hitching offset is almost always less than the length of trailers, therefore the plant is almost always controllable in the neighborhood of the origin.

For a second order system the involutivity condition of the set \( \{ ad_{ij} \bar{g} \ldots ad_{ij}^{n-2} \bar{g} \} \) is always satisfied. This means that a diffeomorphic state transformation \( \bar{z}(\bar{x}) \) for the system (2), (5) theoretically exists. The first element of the state transformation vector is found by solving the system of \( n-1 \) homogeneous partial differential equations \( \frac{\partial z_1}{\partial x_1}(-q_3 \sin x_1 \tan x_2 - q_4) + \frac{\partial z_1}{\partial x_2} \left( q_4 - \frac{q_5}{\cos x_2} \right) = 0 \) and the flat output can’t be found. If we assume that \( q_{24} \) is the momentary value of \( q_2 \sin x_1 \tan x_2 + q_4 \), then the instantaneously approximated equation \( \frac{\partial z_1}{\partial x_1}(-q_{24}) + \frac{\partial z_1}{\partial x_2} \left( q_4 - \frac{q_5}{\cos x_2} \right) = 0 \), results for the expression of the flat output can easily be found:

\[
z_1(\bar{x}) = \frac{2q_{24}q_3}{q_4 \sqrt{q_5^2 - q_4^2}} \arctan \left( \frac{q_5 + q_4 \tan x_2}{\sqrt{q_5^2 - q_4^2}} \right) + \frac{-2q_{24}q_3}{q_4} x_2 - x_1 \quad \text{for } |q_5| > |q_4| \text{ or } l_1 > d_1
\]

(7)

Based upon this, the next elements of the state transformation can be derived as \( z_i = L_i^{-1} z_{i-1} = L_i z_{i-1}, i = 2 \ldots n \) [13]:

\[
z_2(\bar{x}) = q_1 s_1 - q_3 \tan x_2 + \frac{q_3q_{24}q_2}{q_4c_2 - q_5} - \frac{q_{24}}{q_4} x_2
\]

(8)

The calculation of the input transformation also does not cause too many difficulties \( \alpha(\bar{x}) = L_i^{-1} z_1 = L_i z_n, \beta(\bar{x}) = L_i L_i^{-1} z_1 = L_i^2 z_1 \) [13]:

\[
\begin{align*}
\alpha(\bar{x}) &= q_1 c_1 (q_3 \tan x_2 - q_1 s_1) + \\
&\quad - \frac{q_{24} (q_4 - q_5 c_2)}{(q_4c_2 - q_5)^2} - \frac{1}{c_2^2} \frac{q_5^2}{c_2} \tan x_2, \\
\beta(\bar{x}) &= -q_1 q_3 c_1 + \\
&\quad + \frac{q_{24} (q_4 - q_5 c_2)}{(q_4c_2 - q_5)^2} - \frac{1}{c_2^2} \frac{q_3 q_4 c_2 - q_5}{c_2} \tan x_2
\end{align*}
\]

(9)

(10)

With \( |q_5| < |q_4| \) or \( l_1 < |d_1| \), although this is not typical for truck and trailer combinations, the flat output for the approximated system also exists

\[
z_1(\bar{x}) = \frac{-2q_{24}q_3}{q_4 \sqrt{q_5^2 - q_4^2}} \arctanh \left( \frac{q_5 + q_4 \tan x_2}{\sqrt{q_5^2 - q_4^2}} \right) + \frac{-2q_{24}q_3}{q_4} x_2 - x_1
\]

(11)

This leads to the same results for the functions \( z_2(\bar{x}), \alpha(\bar{x}) \) and \( \beta(\bar{x}) \).
D. Control law

Considering the substitution \[ \bar{x} \] the nonlinear control law [13]

\[
\begin{align*}
u &= \frac{1}{\beta(\bar{x})} (v - \alpha(\bar{x})) \\
&= \frac{1}{\beta(\bar{x})} \left( \dot{z}_{id} + T (\ddot{z}_d - \bar{x}) - \alpha(\bar{x}) \right) \\
&= \frac{1}{\beta(\bar{x})} \left( \dot{z}_{id} + b_1 (\ddot{z}_d - \dot{z}_1) + b_0 (\dddot{z}_d) \right) \\
\phi_0 &= \arctan ( u )
\end{align*}
\]

can be represented in a form of two loops, comprising the plant, see Fig. 8.

1) linearization loop with input \( v \) (according to (5)), nonlinear functions \( \beta^{-1}(\bar{x}) \) and \( \arctan \) in the feedback path, as well as \( \alpha(\bar{x}) \) in the feedback path;

2) pole-placement loop with the linear gain \( \tilde{b} = [b_0 \ b_1]^T \) according to the state \( \tilde{z} = [z_1(\bar{x}) \ z_2(\bar{x})]^T \) with references \( \dot{z}_{d} = [\dot{z}_{1d}(\tilde{x}_d) \ \ddot{z}_{2d}(\tilde{x}_d)]^T \) and \( \dddot{z}_{1d} \).

When choosing a stable characteristic polynomial \( N(s) = s^2 + b_1 s + b_0 \), the control law (12) will provide local stability of the closed system, asymptotic convergence to zero of the tracking error \( e(t) = \tilde{z}_d(t) - \tilde{z}(t) \) and the boundedness of all the state variables [13]. In this case transient processes of the flat output of the plant will be described by a linear transfer function \( G(s) = \frac{1}{N(s)} \) [13]. The coefficients \( b_0 \) and \( b_1 \) of the polynomial \( N(s) \) must be chosen based on the desired performance of the closed system as well on the performance of the steering servo.

E. Control scheme

The tracking controller requires smooth reference signals \( z_d, \dot{z}_d, \ldots, \dddot{z}_d \). By a common flatness-based technique they are generated in \( \tilde{z} \) after transformation of the reference signals \( \tilde{x}_d \) to \( \tilde{z}_d \). Additionally, to compensate tracking errors occurring under the influence of model uncertainties and disturbances the measured system output \( \tilde{x}_b \) has to be compared with the output of the nonlinear system model \( \tilde{x}_m \), calculated from the \( \tilde{z}_m \) by the inverse transformation. Here two problems arise: the state transformation is only known for the approximated system, additionally the analytic solution of (7) and (8) is necessary whose derivation is a hard task.

A model-reference control scheme was chosen for this application, comprising a reference-model control loop and a disturbance control loop, see Fig. 4. The reference-model control loop contains the instantaneously approximated nonlinear system model, which is exactly feedback linearized by (9) and (10). The output of the model-reference control loop is \( \tilde{x}_m \), which can thus be obtained without an inverse transformation. The state vector \( \tilde{z}_m \) serves as reference vector for the disturbance control loop, while the linearized control \( v_m \) is passed as a feedforward signal into the linearized part of the disturbance control loop. Due to the feedforward action the stability of the control loops is independent of each other and their controllers can be designed independently for their specific tasks.

As the model control loop is disturbance free, its controller was designed to provide good tracking performance by defining the polynomial \( N_m(s) \) as a second order binomial polynomial. In the absence of disturbances the feedforward control from the model loop would drive the output of the real plant \( \bar{x} \) along the trajectories of the instantaneous approximated model \( \tilde{x}_m \), because both models have the same steady state in \( \bar{x} \). Since the main tracking control is performed in the model loop, the disturbance controller can be designed for good disturbance rejection as its role is to suppress disturbances acting from the environment or as the result of approximation errors. Therefore, the polynomial \( N_m(s) \) was parameterized using maximal possible dynamics under the given dynamics of the steering servo.

However both models, the full model and the approximated model, possess the same steady state in in \( \bar{x} \), but due to the approximated nature of the state transformation system model, the steady states in \( \tilde{x} \) are not equal. To adjust for the incorrect state transformation, the following method was applied. From (8) the value for \( z_{2d} \) is determined for \( z_{2d} = 0 \) and the reference value of \( x_{1d} \). This calculation could have been evaluated analytically with much effort, but we have chosen to solve it numerically in closed loop in the reference generator, see Fig. 4. Based upon the value of \( x_{2d} \) and the reference value of \( x_{1d} \), the value of \( z_{2d} \) was calculated according to (7). The reference vector \( \dddot{z}_m \), which was derived this way, is used as reference for the reference-model control loop. Actually, this ensures the compliance with the steady state compatibility condition using control engineering techniques, similar to the geometric approach taken by Bolzern et al. [8].

To guarantee correct steady states in spite of parametric uncertainties of the plant model, in the last step of designing the model-reference control scheme an overlaying PI-controller was introduced at the input of the disturbance control loop, see Fig. 4.

F. Simulation results

The computer simulation was completed for the demonstrator vehicle at the Institute of Control Engineering with the following parameters: \( l_q = 0.84 m, l_1 = 0.6 m, l_2 = 0.9 m \), \( d_1 = -0.4 m \), \( \phi_{0_{\max}} = \pi / 6 \) and the time constant of the steering servo \( T_{servo} = 0.2 s \).

As the polynomial \( N_m(s) \) in the control law (12) of the reference-model control loop a second-order polynomial was used with the characteristic frequency \( \omega_{0_{m}} = (1,35 s^{-1}) \) and \( b_{2m} = 2 \omega_{0_{m}}, b_{3m} = \omega_{0_{m}}^2 \). The corresponding parameters for the disturbance control loop are given \( \omega_{0_{s}} = (2T_{servo})^{-1} \) and \( b_{1s} = 2 \omega_{0_{s}}, b_{0s} = \omega_{0_{s}}^2 \), with normalized damping factor \( D \).

To demonstrate the robustness of the approach, vehicle parameters with a random variation of up to \( \pm 10\% \) were used in the designed nonlinear controller. Simulations of the reversing process have shown the high quality of the control for TTCs with negative and positive hitching offset, see the system states and the control efforts in Fig. 5 and Fig. 6.
At \( \lambda = 0 \) the system was subjected to a stepwise change of the reference for angle \( \phi_2 \), which defines the desired path curvature of the combination during reversing. The reference value \( \phi_{1d} \) is continually calculated from the steady state compatibility condition. As can be seen in Fig. 5 (left) and Fig. 6 (left), the desired value \( \phi_{2d} \) is met well in the approximated and the full plant model. Due to the deviation between the approximated and the full plant model the desired value \( \phi_{2d} \) is achieved with differing values for \( \phi_{1m} \) and \( \phi_{1s} \). For this reason, the corresponding steering angles \( \phi_{0m} \) and \( \phi_{0s} \) are also not equal, see Fig. 5 (right) and Fig. 6 (right). At \( \lambda = 6m \) a disturbance is included in form of a stepwise reduction of the steering angle by 10% of the maximal value, which simulates a lateral slip effect. It can be seen that this disturbance is suppressed effectively. Although a part of the controller relies on a simplification of the nonlinear plant model, it was shown that the designed controller allows to effectively steer the motion of TTC with an arbitrary offset of the first joint and in both directions of motion. The presented approach was also implemented and successfully validated on the demonstrator vehicle, see Fig. 7 and webpage [14].

III. CONCLUSION

This paper presented the design of a stabilization and tracking controller for a tractor car with two semi-trailers and off-axle trailer hitching, whose kinematic model is not exactly input-state linearizable. The designed nonlinear control law is based on input-state linearization of the instantaneously approximated kinematic model, which can be exactly linearized and whose behavior provides an accurate performance. Despite the efforts of simplification, the designed nonlinear stabilization and tracking law is universal regarding the hitching offset and able to prevent the loss of stability in the closed system while reversing. The presented approach can also be applied to other types of truck and trailer combinations.
Fig. 5. State trajectories (left) and control efforts (right) in plant and model loop of a system with negative hitching offset \( d_1 = -0.4m \) as a consequence of a step input in the reference and in the disturbance signal.

Fig. 6. State trajectories (left) and control efforts (right) in plant and model loop of a system with positive hitching offset \( d_1 = +0.2m \) as a consequence of a step input in the reference and in the disturbance signal.

Fig. 7. Vehicle used to validate the designed control algorithms.

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